Multiwinner Voting with Admissible Sets

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Contents

SRIP 2019 Report on Multiwinner Voting with Admissible Sets

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Abstract

We will consider the problem of multiwinner voting, in which the task is to identify a winning subset of candidates based on a collection of votes that express preferences over all available alternatives. Most literature on multiwinner voting focuses on restricting the size of the output committee. In recent work, [16] consider the problem of finding winning subsets that satisfy additional desirable properties, which they formalize using the notion of *admissible sets*. In our work, we propose to continue this line of study: specifically, we will consider algorithmic questions along the lines of approximation and restricted domains given the existing hardness results in the literature.

Keywords and phrases Computational Social Choice, Approval Ballots, Multiwinner Voting, Dichotomous Preferences

1 Introduction

Approval based multiwinner voting aims to select a subset of candidates, often called committee (or winners), from a class of admissible sets (subsets of candidates) based on the approval ballots of voters. Approval ballots are used in a wide range of areas such as recommendation systems, political elections, etc. Till now, most of the research in approval based multiwinner voting dealt with selecting a committee of cardinality exactly k. [3, 6] Such voting rules are called k-committee selection rules. The setting is quite practical in scenarios where the number of winning candidates is predefined. But, the relation between the candidates is ignored while determining the winners of the committee. For example, if there is a bad blood between two candidates but both are very highly approved by the voters, then the performance of the selected committee can go a notch down because of the irreconcilable relation between the two candidates. In this report, we present multiwinner rules with admissible sets being represented with respect to a graph where candidates are considered as vertices, and edges indicate the relations between candidates.

Let C be a set of candidates. In our setting, we are additionally given a graph G = (C,A). The goal is to find a committee that has some combinatorial property, e.g., the subgraph induced by the committee is connected, is an independent set, etc. When the combinatorial property is "the subgraph induced by the committee excludes all candidates with bad relation", we have the independent committee selection rule. In this selection rule, we have to consider all the candidates without any edge between them, i.e., we define a bad relation between two candidates as an edge between those two nodes. We first investigate the question of how efficiently an optimal committee can be calculated in this setting, i.e., the complexity of the winner determination problem. We particularly focus on approval voting (AV), net-approval voting (NAV), proportional approval voting (PAV), Chamberlin-Courant approval voting (CCAV), satisfaction approval voting (SAV), and net-SAV (NSAV) in our setting 1, aiming to reveal how different combinatorial restrictions on admissible sets shape the complexity of winner determination for these rules.

We first studied the universal admissible sets, i.e., every subset of candidates is an admissible set. Then, we study connected admissible sets, i.e., subsets that induce connected graphs, and bounded radius admissible sets, i.e., subsets that induce graphs of bounded radius. In this two cases, winner determination is NP hard for NSAV and NAV, and polynomial-time solvable for other rules. Moreover, we also study independent admissible sets, i.e., the selected committee should induce an independent set in the associated graph G = (C,A). For PAV and CCAV, the NP-hardness even holds when the associated graph is a path (in this case there is more than one vote). Else, it is NP Hard for even a single vote. This results have been presented by Yang et al. in their recent studies. [16] We show FPT results using Steiner Tree of the associated graph for connected graph property.

We would like to further restrict the structure of the underlying graph imposed on the candidates by working with preferences from restricted domains. We explore a number of domain restrictions for dichotomous preferences that build on the same intuition as the concepts of single-peakedness and single-crossingness. We analyze the relationships among these restricted preference domains, and discuss the complexity of detecting whether a given dichotomous profile belongs to one of these domains. We also discuss the algorithmic complexity for the various voting rules under some of these domain restrictions. We would also like to extend this study to the model where votes are expressed as rankings, rather than as approval ballots, over the set of candidates.

2 Literature Review

The work of Yang and Wang [16] discusses approval-based multiwinner voting to select a set of candidates from a collection of subsets which they refer to as “admissible sets”. This generalizes the typical setting in which the problem is studied, which addresses the question of finding committees of size k, which can be viewed as the setting where every set of size k is admissible while others are not. The drawback of the default setting is that it does not account for the relation between the candidates while selecting them. The admissible sets setting allows us to encode a richer set of constraints on the winning committees. In particular, the study in [16] represents the election as a graph in which the vertices are taken as candidates and the edges represent the relationship between them, and admissible sets are vertex subsets that might be required to induce certain graph properties (such as being independent or being connected).

The main question addressed is the complexity of winner determination when the committees are required to belong to the family of admissible sets. The authors also consider issues of strategyproofness of various multiwinner rules in this setting. The scoring rules which have been used for analysing the multiwinner voting are: Approval Voting (AV), Net Approval Voting (NAV), Proportional Approval Voting (PAV), Chamberlin-Courant Approval voting (CCAV), Satisfaction Approval Voting (SAV), and Net SAV (NSAV). Further, the graph properties used to encode admissible sets that are used in the paper are the following:

- Universal: All graphs are included. No relation between the candidates is taken into consideration, also no limit on the number of candidates being selected.

- Fixed-size: All graphs containing only k vertices. Used for analysis in some of the previous works. Again, no relation between the candidates is taken into consideration.
- Independent: All graphs without edges. When there is an incompatible relation between two candidates.
- Connected: All graphs connected by edges. Exactly opposite of Independent property.
- Bounded Radius: All graphs whose radius is at most d. There is at least one vertex in the graph which is at a minimum distance d' from the rest of the vertices. This property can be used to solve the example taken in the introduction section of the paper.

2.1 Related Work

Multiwinner voting is a classical topic that is widely addressed in the literature, while the notion of a graph associated with the candidates of an election (or voters in an election) is also considered in many situations. The following is a brief summary of related work.

- N. Talmon : In this paper, the author has studied a generalization of proportional representation of CCAV to select a k-sized committee [15]. The paper discusses about the relation between each voter and how it can affect the final result. A graph is used for showing the relation between the voters with each vertex representing a voter.
- Aziz et al. (2015) : In this paper, the authors have discussed three rules for multi-winner approval voting namely: PAV, SAV, RAV [3]. They have used fixed size set of approval ballots for winner determination. Further, they have also analysed strategyproofness for AV under the same restrictions on the admissible sets.
- Betzler et al. : Proof for winner determination using CCAV multiwinner rule is NP-Hard is discussed in this paper [4]. They further try to find methods by which the problem can be solved effectively. They consider minimizing the maximal misinterpretation.
- D. Marc Kilgour : In this paper, a subclass of multi winner elections, variable number of winners (VNW) elections is discussed [13]. Many methods of counting approval ballots appropriate to VNW elections are reviewed and illustrated in this paper.
- Faliszewski et al. : An approval based multiwinner model of election was considered in this paper, in which universal admissible set was taken for winner determination [11].
- Bredereck et al. : This paper discusses a performance-based measure of the quality of the committee [5]. They have considered some restrictions on the candidates, without qualifying for which, the candidate won't be able to be a part of any winning committee for the election regardless of her individual performance amongst the voters.
- Celis et al. : In this paper, it is discussed that while selecting a committee, if candidates have some special attributes, there arises a need for optimizing a multiwinner voting rule, as the voting rule may under or over represent in its selection [6].
- Aziz et al. : This paper answers one of the 'future problems' for the fixed admissible set property. This paper discusses the proportionality property (an axiom) named 'justified representation' [1]. This axiom states that if a large enough group of voters exhibits agreement by supporting the same candidate, then at least one voter in this group has an approved candidate in the winning committee.

3 Preliminaries

An election is a tuple E = (C,V) where C is a set of candidates and V is a multiset of votes, each of which is a subset of C. For a vote v ∈ V and a candidate c ∈ C, we say v approves c if and only if c ∈ v. Let 2^C be the power set of C. A multiwinner rule ϕ is a function that assigns to each election (C,V) a subset ϕ(C,V) $\in A_C$, where $A_C \subseteq 2^C$ is the co-domain and each element of A_C is called an admissible set. The elements in ϕ(C,V) are called the winners of (C,V) with respect to ϕ.

Table 1 Scoring Functions

multiwinner rules	**scoring functions** $f(v, w)$
Approval Voting (AV)	$\|v \cap w\|$
Proportional Approval Voting (PAV)	$1 + \frac{1}{2} + ... + \frac{1}{\|v \cap w\|}$
Chamberlin-Courant Approval Voting (CCAV) [7]	1 if $v \cap w \neq \phi$ 0 if $v \cap w = \phi$
Satisfaction Approval Voting (SAV)	$\frac{\|v \cap w\|}{\|v\|}$
Net-SAV (NSAV)	$\frac{\|v \cap w\|}{\|v\|} - \frac{\|v \setminus w\|}{\|C\|-\|v\|}$
Net-Approval (NAV)	$\|v \cap w\| - \|v \setminus w\|$

Consider the multiwinner voting rules defined based on the scoring function f: $2^C \times 2^C \rightarrow \mathbb{R}$. For a subset C' $\subseteq$ C, the score of C' in (C, V), with respect to f in $\sum_{v \in V} f(v,C')$. The multiwinner rule ϕ selects an admissible set C' $\in A_C$ maximizing $\sum_{v \in V} f(v,C')$. Table 1 summarizes some of the well known multiwinner rules and their scoring functions. As the number of admissible sets can be exponential in the number of candidates, a compact representation based on graph property has been proposed by Yang et al. [16] A graph property $\mathcal{G}$ is a class of graphs. Fixing a graph property $\mathcal{G}$, the class of admissible sets $A_C^{\mathcal{G}}$ consists of all C' $\subseteq$ C such that G|C'| $\in \mathcal{G}$, where G|C'| is the subgraph induced by C'. Graph properties studied are as follows:

1. **Universal** This property consists of all graphs, i.e., every subset of C is an admissible set.
2. **Fixed-sized** This is so far the most widely studied property. Particularly,it consists of all graphs of exactly k vertices.
3. **Independent** The property consists of all graphs without edges. In some cases,there may exist irreconcilable conflicts between two candidates, thus, an edge between such two candidates indicates the existence of an irreconcilable conflict between them.
4. **Connected** This property consists of all connected graphs. Different from the independent property, the edge between two candidates can be a positive indication (e.g., an edge means that they can cooperate efficiently, or they can communicate directly).
5. **Bounded Radius** This property consists of all graphs with radius at most d, where d is a constant. The distance between two vertices is the length of a shortest path between them.

For a graph property $\mathcal{G}$ and a multiwinner rule ϕ_f, the following problem is studied:

Winner Determination With Restricted Admissible Sets (WD-$(\mathcal{G}, \phi_f)$)

Input: An election (C,V), a graph G = (C,A), and a rational number r.

Question: $\exists w \in A_C^{\mathcal{G}}$ such that $\sum_{vinV} f(v, w) \geq r$?

Let C = $c_1, c_2, ..., c_m$ be a finite set of candidates. A partial order $\succ$ over C is a reflexive, antisymmetric and transitive binary relation on C; a partial order $\succ$ is said to be total if for each c, d $\in$ C we have c $\succ$ d or d $\succ$ c. A partial order $\succ$ over C is a dichotomous weak order if C can be partitioned in to two disjoint sets C^+ and C_- (one of which may be empty) so that c $\succ$ d for each c $\in C^+$, d $\in C^-$ and the candidates within C^+ and C^- are in comparable under $\succ$.

An approval vote on C is an arbitrary subset of C. We say that an approval vote v is trivial if $v = \phi$ or v = C. A dichotomous profile P = $v_1, v_2, ..., v_n$ is a list of n approval votes; we will refer to v_i as the vote of voter i. Therefore, $\bar{v_i}$ = C$\backslash v_i$

(a) Single Peakedness **(b)** Single Crossing

Figure 1 Preferences in Restricted Domain [10]

Let $\lhd$ be a total order over C. A total order $\succ$ over C is said to be single-peaked with respect to $\lhd$ if for any triple of candidates $a, b, c \in C$ with $a \lhd b \lhd c$ or $c \lhd b \lhd a$ it holds that $a \succ b$ implies $b \succ c$. A profile P of total orders over C is said to be single-peaked if there exists a total order $\lhd$ over C such that all orders in P are single-peaked with respect to $\lhd$

A profile $P = (\succ_1, \succ_2, ..., \succ_n)$ of total orders over C is said to be single-crossing with respect to the given order of votes if for every pair of candidates $a, b \in C$ such that $a \succ_1 b$ all votes where a is preferred to b precede all votes where b is preferred to a. P is single-crossing if the votes in P can be permuted so that it becomes single-crossing with respect to the resulting order of votes.

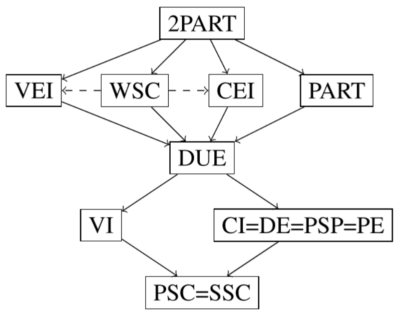
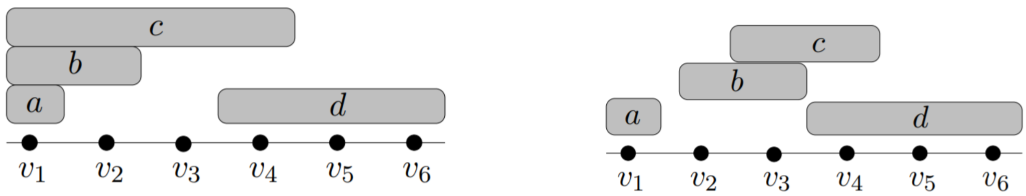


Figure 2 Relation between notions of structures [9]

Few dichotomous constraints need to introduced first before indulging further. Most of these constraints can be divided into two basic groups: those that are based on ordering voters and/or candidates on the line and requiring the votes to respect this order (this includes VEI, VI, CEI, CI, DE, and DUE), and those that are based on viewing votes as weak orders and asking if there is a single-peaked / single-crossing / 1Euclidean profile of total orders that refines the given profile (this includes PSP, PSC, and PE)

1. **2-partition (2PART):** We say that P satisfies 2PART if P contains only two distinct votes v, v', and $v \cap v' = \phi$, $v \cup v'$ = C.
2. **Voter Extremal Interval (VEI):** We say that P satisfies VEI if the voters in P can be reordered so that for every candidate c the voters that approve c form a prefix or a suffix of the ordering. Equivalently, both the voters who approve c and the voters who disapprove c form an interval of that ordering.
3. **Voter Interval (VI):** We say that P satisfies VI if the voters in P can be reordered so that for every candidate c the voters that approve c form an interval of that ordering.
4. **Candidate Extremal Interval (CEI):** We say that P satisfies CEI if candidates in C can be ordered so that each of the sets v_i forms a prefix or a suffix of that ordering. Equivalently, both $\bar{v_i}$ and v_i form an interval of that ordering.
5. **Candidate Interval (CI):** We say that P satisfies CI if candidates in C can be ordered so that each of the sets v_i forms an interval of that ordering.
6. **Weakly single-crossing (WSC):** We say that P satisfies WSC if the voters in P can be reordered so that for each pair of candidates $a, b, c \in$ C it holds that each of the vote sets $V_1 = v_i : a \in v_i, b \notin v_i$, $V_2 = v_i : a \notin v_i, b \in v_i$, $V_3 = v \in P : v \notin V_1 \cup V_2$forms an interval of this ordering, with V_3 appearing between V_1 and V_2.



(a) Voter Extremal Interval (VEI) **(b)** Voter Interval (VI)

Figure 3 When Voters are embedded on real axis [9]

(a) Candidate Extremal Interval (CEI) **(b)** Candidate Interval (CI)

Figure 4 When Candidates are embedded on real axis [9]

4 Theorems and Lemmas

It is known that for fixed-sized admissible sets, winner determination for PAV and CCAV is NP-hard [3, 4] while the for the rest, it is polynomial time solvable. Winner determination is polynomial time solvable for all multiwinner rules for universal admissible sets [6, 2, 16].

Below are some of the theorems which have been studied by Yang et al. in their paper "Multiwinner Voting with Restricted Admissible Sets" [16].

▶ **Theorem 1.** *For $\mathcal{G}$ being the universal property and $\phi_f \in$ AV, PAV, CCAV, SAV, NSAV, NAV, WD-($\mathcal{G}, \phi_f$) is a polynomial time solvable [16].*

Proof. For AV, PAV, CCAV, SAV voting rules, the the set C of all candidates is always an optimal committee, as there is no such negative score. So, for these rules we need only to check if the score of C is at least r. Now we consider NSAV. For each candidate c, we define

$$g(c) = \sum_{v \in V, c \in v} \frac{1}{|v|} - \sum_{v \in V, c \notin v} \frac{1}{|C| - |v|}$$

Let C' = $c \in$ C | $g(c) > 0$. If C' $\neq \phi$, we return Yes if and only if the score of C' is at least r. Otherwise, we return Yes if and only if there is a candidate c such that $g(c) \geq$ r. ◀

For connected admissible sets, we also obtain polynomial time solvability results for CCAV, PAV, AV, and SAV, but obtain NP-hardness results for NAV and NSAV even if there is only one vote. This is because that for the first four rules, adding a candidate to a committee never decreases the score of the committee. Hence, there must be an optimal committee which induces a connected component. However, this is not the case for NAV and NSAV. This is so because we need to take care of the negative score that might arise in some cases so as to make the graph a connected graph, i.e., whose role is to connect the candidates with the positive scores.

▶ **Theorem 2.** *For $\mathcal{G}$ being the connected property and $\phi_f \in$ NAV, NSAV, WD-($\mathcal{G}$, ϕ_f) is NP-hard, even if there is only one vote [16].*

RESTRICTED EXACT COVER BY 3-SETS (RX3C) [12]

Input: A finite set U = $u_1, u_2, ..., u_{3k}$ and a collection S = $s_1, s_2, ..., s_{3k}$ of 3-subsets of U s.t. every $u \in$ U occurs in exactly three subsets of S.

Question: $\exists S' \subseteq$ S such that $|S'| = k$ and every $u \in$ U occurs in exactly one subset of S'?

Proof. First, we will give construction of the instance. Given an RX3C instance (U,S) where|U| = |S| = $3k$, we create an instance of WD-($\mathcal{G}, \phi_f$) as follows. For each $u \in$ U, we create a candidate $c(u)$. For each $s \in$ S, we create a candidate $c(s)$. Let C(U) = $c(u)|u \in U$, C(s) = $c(s)|s \in S$. In addition, we also introduce a candidate b. Hence, C = C(U)∪C(S)∪b. In the graph G = (C,A), we create an edge between b and every $c(s)$, where $s \in$ S. Additionally, for every $c(s)$, $s \in$ S, and every $c(u)$, $u \in$ U, we create an edge between them if and only if $u \in s$. Moreover, we create one vote v which approves all candidates in C(U) and disapproves all other candidates, i.e., v = C(U). Finally, we set r = $2k - 1$.

Now we assume that there is an exact 3-set cover S' ⊆ S. Let $w = c(s)|s \in S \cup C(U) \cup b$. Therefore, size of the admissible set |w| = $4k$ - 1. Also, w induces a connected graph in G. NAV Score will be $|v \cap w| - |w \backslash v|$, which is equal to ($3k$ - ($2k + 1$) = r. Now we prove the correctness for the opposite direction. Now we assume the score of the committee w to be atleast r, i.e., $2k - 1$. Let x = |$w \cap C(U)$| and y = |$w \cap C(S)$|. Now, we assume that b is in the winning committee w. We will prove this using contradiction. Let us assume that $b \notin w$. Therefore, to make the graph a connected subgraph, it should satisfy $y > \frac{x-1}{2}$. Hence, the score of the winning committee w will be x - y $\leq \frac{3k+1}{2}$ which is less that $2k$ - 1. Thus, our assumption is wrong. b is in w. To make the graph a connected graph, $y\ geq \frac{x}{3}$. Therefore, score of w is x - y - 1 $\leq \frac{2x}{3} - 1$. If x = $3k$, score will be atmost $2k - 1$. Therefore, to have a winner determination, x = C(U) and y = C(S). ◀

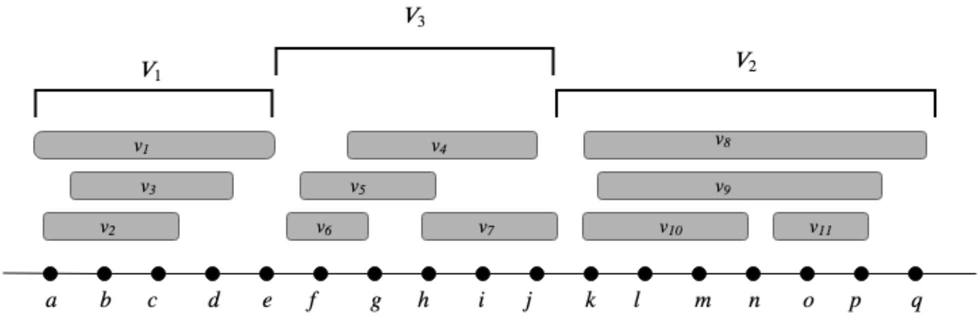


Figure 5 Weakly Single Crossing (WSC)

▶ **Theorem 3.** *For $\mathcal{G}$ being class of graphs with radius atmost 2 and $\phi_f \in$ NAV, NSAV, WD-($\mathcal{G}$, ϕ_f) is NP-hard, even if there is only one vote [16].*

Proof. In the WD-($\mathcal{G}, \phi_f$) instance constructed in the proof of Theorem 2, every optimal committee includes b and k candidates in C(S) to connect the candidates in C(U). The induced subgraph has radius 2. This directly gives us the above result. ◀

▶ **Theorem 4.** *For $\mathcal{G}$ being class of graphs with radius atmost d and $\phi_f \in$ AV, PAV, SAV, CCAV, WD-($\mathcal{G}$, ϕ_f) is polynomial-time solvable [16].*

▶ **Theorem 5.** *For $\mathcal{G}$ being the independent property and $\phi_f \in$ PAV, CCAV, WD-($\mathcal{G}, \phi_f$) is NP-hard, even if the associated graph is a path and every voter approves only two candidates [16].*

> ALMOST 2-SAT
>
> **Input:** A set $X = \{x_1, x_2, ..., x_m\}$ of Boolean variables, a set $CL = \{cl_1, cl_2, ..., cl_n, l\}$ of clauses each of which consists of exactly two literals of variables in X, and a positive integer l.
>
> **Question:** Is there a truth assignment $\delta : X \rightarrow \{0, 1\}$ which satisfies at least l clauses in CL? Here, a clause $cl \in$ CL is satisfied by δ if there exists a literal $x \in cl$ such that $\delta(x) = 1$ or a literal $\bar{x} \in cl$ such that $\delta(x) = 0$

Proof. Let $X = x_1, x_2, ..., x_m$ and $cl_1, cl_2, ..., cl_n, l$ be an instance of the ALMOST 2-SAT problem. Construction is done as follows. For each $x \in X$, we create two candidates $c(x)$ and $c(\bar{x})$. Also, we introduce additional $m-1$ dummy candidates $c_1, c_2, ..., c_{m-1}$. For every $x_i \in$ X, $1 \leq i \leq m$, there is an edge between $c(x_i)$ and $c(\bar{x_i})$. In addition, for every $c_i, 1 \leq i \leq m-1$, there is an edge between c_i and $c(x_i)$, and an edge between c_i and $c(\overline{x_{i+1}})$. For each $cl \in$ CL, we create three votes $v(cl, 1)$, $v(cl, 2)$ and $v(cl, 3)$. The votes are as follows: $v(cl, 1)$ and

$v(cl, 2)$ are the same and they approve $c(y)$ for every literal $y \in cl$ and $v(cl, 3)$ approves $c(\bar{y})$ for every literal $y \in cl$. Also, set r = $\frac{3}{2}.(l + n)$

First, we assume that there exists at least l clauses in CL. Let CL' be the set of all clauses that are satisfied by δ, thus, |CL'| $\geq l$. Therefore, w is a set of candidates such that they follow the independent property, The set w can be represented as follows:

$$w = \{c(x) \mid x \in X, \delta(x) = 1\} \cup \{c(\bar{x}) \mid x \in X, \delta(x) = 0\}$$

Observe that for every clause $cl \in$ CL', either both literals in cl are true or exactly one of them is true with respect to δ. Due to the construction, in the former case, both approved candidates of $v(cl, 1)$ and $v(cl, 2)$ are in w, and none of the approved candidates of $v(cl, 3)$ is in w. Therefore, the score for them will be $f(v(cl, 1)) = f(v(cl, 2)) = \frac{3}{2}$. If exactly one of them is true, then the score will be $f(v(cl, 1)) = f(v(cl, 2)) = f(v(cl, 3)) = 1$. In both cases, we have the total score of the satisfied clause to be 3. Now, we will consider clauses which are not satisfied by the truth assignment δ, $cl \in$ CL\ CL', therefore, both the candidates are approved by $v(cl, 3)$. Score will be $\frac{3}{2}$. Now, the election score is:

$$\begin{aligned}\sum_{v \in V} f(v, w) &= \sum_{cl \in CL'} \left(\sum_{1 \leq i \leq 3} f(v(cl, i), w) \right) + \sum_{cl \in CL \setminus CL'} \left(\sum_{1 \leq i \leq 3} f(v(cl, i), w) \right) \\ &= 3.|CL'| + \frac{3}{2}.|CL \setminus CL'| \geq \frac{3}{2}(l + n) = r\end{aligned}$$

The proof in reverse direction is as follows. There won't be any dummy candidates now. We know that the candidates are independent and we know r. Therefore, it holds exactly one of $c(i)$ and $c(\bar{i})$. If let's say, the winning committee satisfies l' clauses, then the score of the election will be $\frac{3}{2}(l' + n)$. But, this should be at least greater than or equal to r, which is equal to $\frac{3}{2}(l + n)$. Therefore, l' equals l, i.e., it should satisfy l clauses. ◀

5 Our Contributions (Discussion)

It is known that for connected admissible sets, winner determination for NAV and NSAV is NP Hard, even if there is only one voter. [16] We show that winner determination for NAV and NSAV is polynomial time solvable if we restrict the size of the voter to atmost k.

Lemma 1: *For $\mathcal{G}$ being the connected property and $\phi_f \in$ NAV, NSAV, WD-($\mathcal{G}$, ϕ_f) is polynomial time solvable in $2^k 3^k$poly(n), for one vote of size at most k.*

Construction: The winner determination function will select the maximum NAV score for the election. This will be possible when the candidates included in the admissible set from the voter set are maximised and those not in the voter set are minimized. The graph has connected property, so we need to select an admissible set such that it is a connected graph. Thus, we will need to choose minimal number of candidates to make |w ∩ v| a connected graph. This can be achieved by using Steiner Trees. The dynamic programming algorithm for Steiner trees can be implemented with running time $3^{|k|} n^{\mathcal{O}(1)}$ [8].

Though the voting rule is polynomial time solvable in $6^k poly(n)$ for one voter, it can be seen that in case of multiple voters, it gives a time complexity of $\mathcal{O}(c^{nk})$. Thus, it can be seen it is not a very efficient algorithm in case of multiple votes. We can also notice that the restricted preferences cannot be imposed on graph. Thus, a need for efficient algorithms becomes essential.

6 Conclusion

We have thus studied multiwinner voting with different admissible sets by representation using graph properties. We also tried imposing various restricted preferences on the graph. We studied the winner determination complexity for different voting rules on various graph properties. After that, we tried to improve the known results for winner determination complexity of NAV/NSAV Voting rules on the connected graph property by restricting the size of the vote to at-most k. We were able to make the winner complexity to be solvable in polynomial time $\mathcal{O}(6^k poly(n))$ for one vote.

Further, we tried to make it solvable in polynomial time for n voters for NAV/NSAV voting rules on a connected graph. We can try exploring reduction to Red Blue Dominating Sets to try to prove that the above it W-Hard solvable. It is very interesting to see that once we are able to make the voting rule polynomial solvable for n voters, we can try imposing restricted preferences in dichotomous domains such as VEI, CEI or WSC on the given candidate or voter sets. Further, we can explore the strategyproofness results for the restricted admissible sets. We plan to futher pursue this in two major directions: the first is by restricting the structure of the underlying graph imposed on the candidates, and the second is by working with preferences from restricted domains. We would also like to extend this study to the model where votes are expressed as rankings, rather than as approval ballots, over the set of candidates.

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