

① Determine whether following systems are memoryless, causal, additivity, linearity, memoryless, stability & time-invariance

$$(a) \quad y[n] = e^{x[n]}$$

$$(b) \quad y(t) = x(t) + 3x(t-t_0) - 2y(t-t_0) \quad t_0 > 0$$

$$(c) \quad y(t) = \int_{-\infty}^{2t} x(z) dz$$

$$(d) \quad y(t) = \int_{-\infty}^{\infty} |x(z)| dz$$

② Prove that the system represented by convolution

$$\text{sum} \quad y(t) = \int_{-\infty}^{\infty} x(z) h(t-z) dz$$

is always linear & time invariant. Further derive causality & stability conditions.

③ Given following continuous time signals, determine the Nyquist rate for sampling the signal.

$$① \quad x(t) = \cos 2\pi t + \cos 8\pi t$$

$$② \quad x(t) = \sum_{k=1}^K \cos(\pi k t)$$

$$③ \quad x(t) = e^{j4\pi t}$$

$$④ \quad x(t) = \sum_{k=-N}^N a_k e^{jk 2\pi t}$$

$$⑤ \quad \frac{d^2 x(t)}{dt^2} = -k^2 x(t)$$

④ A signal $x(t) = \cos(2\pi f_0 t)$ is sampled with sampling frequency f_s . If $\hat{x}(t)$ is obtained such that

$$\hat{x}(t) = x(t) \sin(2\pi f_1 t)$$

What should be the value of f_1 such that the signal $\hat{x}(t)$ can be sampled at $2f_s$.

5] Sketch the following signals :-

$$\textcircled{1} \quad x(t) = 2\delta(t+2) - 2\delta(t-2)$$

$$\textcircled{2} \quad x[n] = \cos(2\pi 0.1n) u[n]$$

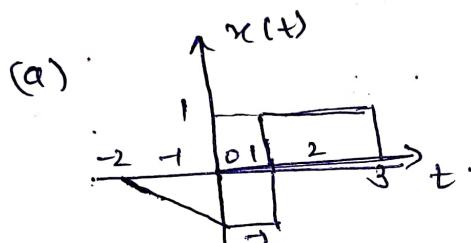
$$\textcircled{3} \quad x[n] = n[u(n+1) - u(n-2)] + n^2 u[n-2]$$

$$\textcircled{4} \quad x(t) = \int_{-\infty}^t u(z) dz.$$

$$\textcircled{5} \quad \mathcal{E}\{u[n]\}, \mathcal{Q}\{u[n]\}$$

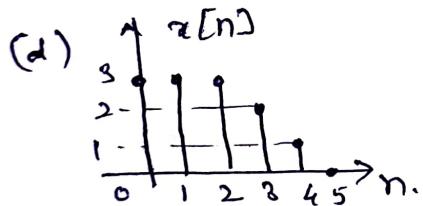
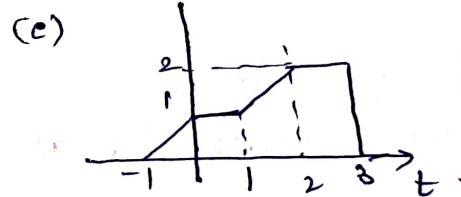
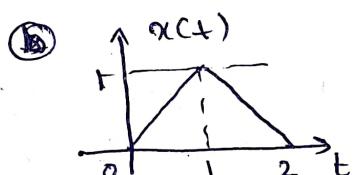
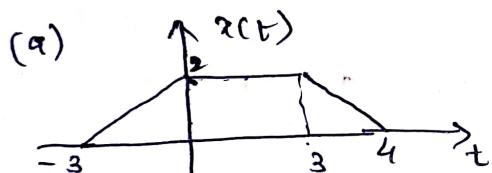
$$\textcircled{6} \quad \mathcal{E}\{x(t)\}, \mathcal{Q}\{x(t)\} \quad x(t) = \delta(t)$$

$$\textcircled{7} \quad \frac{d}{dt} x(t)$$



$$(b) \quad d(t)$$

6] Represent using piecewise representation using ~~unit~~ unit step.



7] Determine if following signals are periodic or not. If periodic determine its period

$$(a) \quad x(t) = \sum_{k=1}^K \cos(kt) \quad k \in \mathbb{Z}$$

$$(b) \quad x(t) = |\cos(\pi n)|$$

$$(c) \quad x[n] = \cos \pi n + \cos^2 \pi n + \cos^3 \pi n$$

$$(d) \quad x[n] = e^{j\pi n/6}$$

$$(e) \quad x[n] = \frac{\sin(\pi n)}{\pi n}$$

$$(f) \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk2\pi t}$$

$$(g) \quad x[n] = e^{j\pi n/12}$$

8] Determine if the following signals are energy or power signals

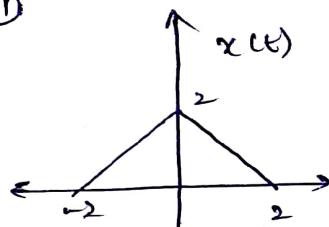
- (a) $x(t) = \max(0, t)$
- (b) $x(t) = \sin(t) \cos(t) u(t)$
- (c) $x(t) = e^{-j(t+4)}$
- (d) $x[n] = 3^n u[-n]$

(e)

9] Convolution.

Perform convolution & verify commutative property of convolution i.e.
 $x(t) * h(t) = h(t) * x(t)$
 $x[n] * h[n] = h[n] * x[n]$

①

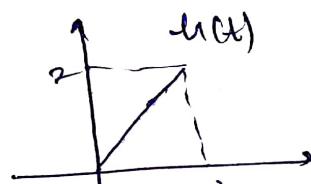
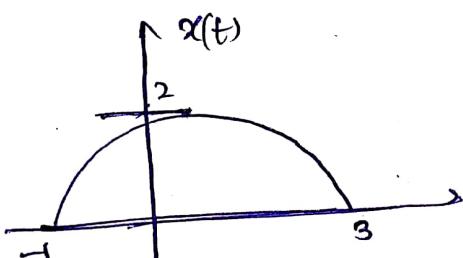


②

$$x(t) = 3^{-t} u(t)$$

$$h(t) = \delta(t+3/2) - \delta(t-3/2)$$

③



④

$$x[n] = \delta(n) + 2\delta[n+1] - \delta[n-3]$$

$$h[n] = 2\delta[n+1] + 2\delta[n-1]$$

⑤

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2] \quad h[n] = u[n+2]$$

⑥

$$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1], \quad h[n] = u[n-1]$$