LAPLACE TRANSFORM.

1] $x(t) = e^{-at} u(t)$ $\quad a > 0$

2] $x(t) = e^{-at} u(t) + e^{-bt} u(t)$ $\quad a, b > 0$

3] $x(t) = e^{-at} u(t) + e^{-bt} u(-t)$ $\quad a, b > 0$

4] $x(t) = (3e^{-4t} - 2e^{-2t}) u(t)$

5] $x(t) = e^{-a|t|}$ $\quad a > 0$

6] $x(t) = t e^{-t} u(t)$

7] $x(t) = t^2 e^{-3t} u(t)$

8] $x(t) = \sin(\Omega_0 t) u(t)$

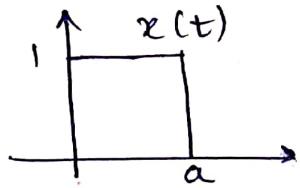
9] $x(t) = \cos(\Omega_0 t) u(t)$

10] $x(t) = e^{-at} \sin(\Omega_0 t) u(t)$ $\quad a > 0$

11] $x(t) = e^{-at} \cos(\Omega_0 t) u(t)$ $\quad a > 0$

12] $x(t) = t e^{-2t} \sin(2t) u(t)$

13]



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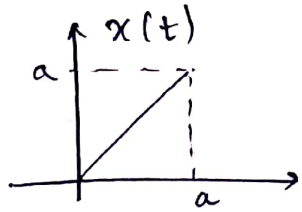

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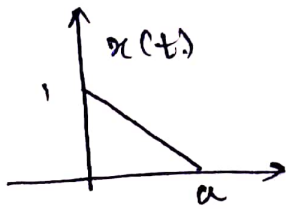
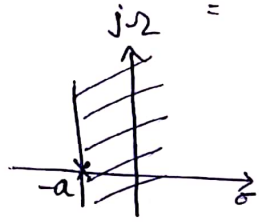


① $x(t) = e^{-at} u(t)$ $\qquad a>0$

$$X(s) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st} dt$$

$$= \int_{0}^{\infty} e^{-(a+s)t} dt = \left. \frac{e^{-(a+s)t}}{-(a+s)} \right|_{0}^{\infty}$$



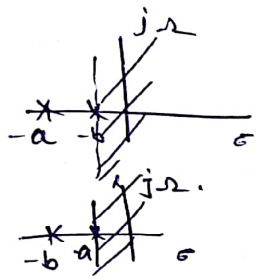
$$X(s) = \frac{1}{s+a} \qquad \text{ROC: } \sigma > -a$$

② $x(t) = e^{-at} u(t) + e^{-bt} u(t)$ $\qquad a, b > 0$

$$X(s) = \frac{1}{s+a} + \frac{1}{s+b}$$

if $a > b$ $\qquad$ ROC:- $\sigma > -b$

if $b > a$ $\qquad$ ROC:- $\sigma > -a$

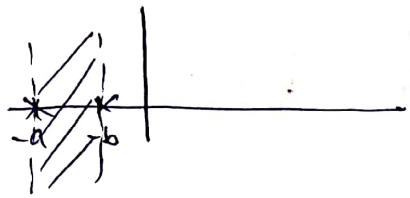


③ $x(t) = e^{-at} u(t) + e^{-bt} u(-t)$

$$= \frac{1}{s+a} - \frac{1}{s-b}$$

$\Rightarrow \sigma > -a$ & $\sigma < -b$

if $a > b$



if $a < b$

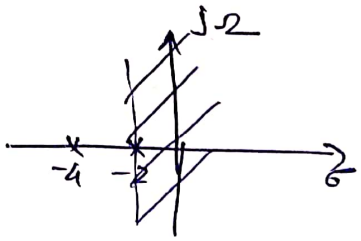
$\Rightarrow$ No ROC overlap $\Rightarrow$ LT doesn't exist.

4] $x(t) = (3e^{-4t} - 2e^{-2t})\,u(t)$

$$X(s) = \frac{3}{s+4} - \frac{2}{s+2}$$

$\sigma > -4$
$\sigma > -2$

$\Rightarrow$ ROC: $\sigma > -2$



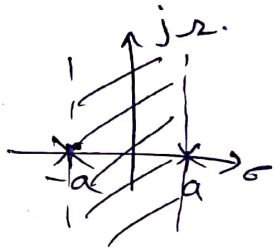
5] $x(t) = e^{-a|t|}$

$= e^{at} \qquad t \leq 0$

$= e^{-at} \qquad t \geq 0$

$$X(s) = \frac{-1}{s-a} + \frac{1}{s+a}$$

$\sigma < a \qquad \sigma > -a$

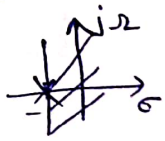


6] $x(t) = t e^{-t} u(t)$

$$\mathcal{L}\{e^{-t}u(t)\} = \frac{1}{s+1}$$

$$X(s) = -\frac{d}{ds}\left(\frac{1}{s+1}\right) = \frac{1}{(s+1)^2}$$

ROC: $\sigma > -1$



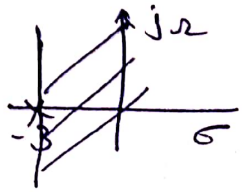
7] $x(t) = t^2 e^{-3t} u(t)$

$$\mathcal{L}\{e^{-3t}u(t)\} = \frac{1}{s+3}.$$

$$\mathcal{L}\{t e^{-3t}u(t)\} = \frac{1}{(s+3)^2}$$

$$X(s) = \frac{-2}{(s+3)^3}$$

ROC: $\sigma > -3$



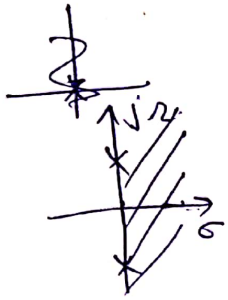
8] $x(t) = \sin(\Omega_0 t)\, u(t)$

$$X(s) = \int_0^\infty \frac{e^{j\Omega_0 t} - e^{-j\Omega_0 t}}{2j} e^{-st}\, dt$$

$$= \frac{1}{2j} \int_0^\infty e^{(j\Omega_0 - s)t} - e^{-(j\Omega_0 + s)t}\, dt$$

$$= \frac{1}{2j} \left[\frac{e^{(j\Omega_0 - s)t}}{-(s - j\Omega_0)} - \frac{e^{-(j\Omega_0 + s)}}{-(s + j\Omega_0)} \right]_0^\infty$$

ROC: $\sigma > 0$



$$= \frac{1}{2j} \left[\frac{1}{s - j\Omega_0} - \frac{1}{s + j\Omega_0} \right]$$

$$\boxed{X(s) = \frac{\Omega_0}{s^2 + \Omega_0^2} \qquad \text{ROC}: \sigma > 0}$$

9] $x(t) = \cos(\Omega_0 t)\, u(t)$

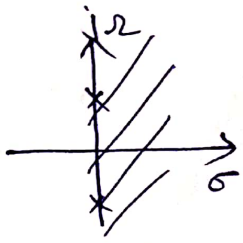
$$X(s) = \int_0^\infty \frac{e^{j\Omega_0 t} + e^{-j\Omega_0 t}}{2} e^{-st}\, dt$$

$$= \frac{1}{2} \int_0^\infty \left(e^{-(s - j\Omega_0)t} + e^{-(s + j\Omega_0)t} \right) dt$$

$$= \frac{1}{2} \left[\frac{e^{-(s - j\Omega_0)t}}{-(s - j\Omega_0)} + \frac{e^{-(s + j\Omega_0)t}}{-(s + j\Omega_0)} \right]_0^\infty$$

$\sigma > 0$

$$\boxed{X(s) = \frac{s}{s^2 + \Omega_0^2} \qquad \text{ROC}: \sigma > 0}$$



10] $x(t) = e^{-at}\sin(\Omega_0 t)\,u(t)$. $a > 0$

$$X(s) = \frac{\Omega_0}{(s+a)^2+\Omega_0^2}$$

ROC: $\sigma > -a$

11] $x(t) = e^{-at}\cos(\Omega_0 t)\,u(t)$ $a > 0$

$$X(s) = \frac{s+a}{(s+a)^2+(\Omega_0)^2}$$

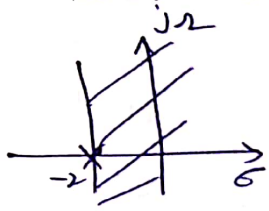
ROC: $\sigma > -a$

12] $x(t) = \cancel{e^{-at}}\; t\, e^{-2t}\sin(2t)\,u(t)$

$$\mathcal{L}\{e^{-2t}\sin 2t\; u(t)\} = \frac{2}{(s+2)^2+4}$$

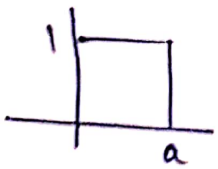
$$X(s) = \frac{+2}{((s+2)^2+4)^2}\; 2(s+2)$$

ROC: $\sigma > -2$



$$= \frac{4(s+2)}{[(s+2)^2+4]^2}$$

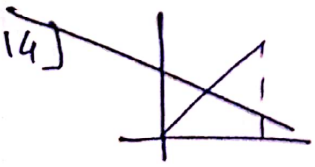
13] $x(t) = u(t) - u(t-a)$

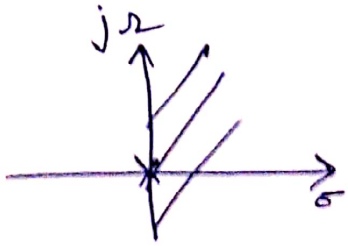


$$X(s) = \frac{1}{s} - \frac{e^{-sa}}{s}$$

$\sigma > 0$

14]





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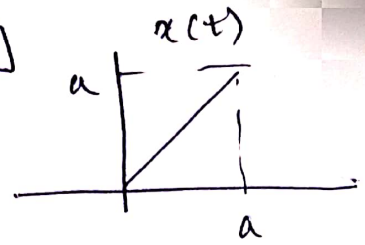


$$x(t) = t\,(u(t) - u(t-a))$$

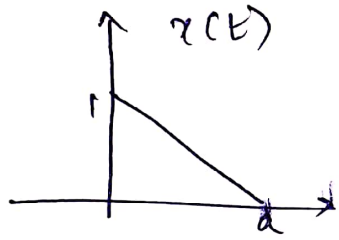
$$\mathcal{L}\{u(t) - u(t-a)\}$$

$$= \frac{1}{s} - \frac{e^{-as}}{s} \qquad \sigma > 0$$

$$\therefore X(s) = \frac{1}{s^2} + \left[\frac{e^{-as} + as\,e^{-as}}{s^2}\right]$$

$$= \frac{1 + e^{-as}(1 + as)}{s^2} \qquad \sigma > 0$$

15]



0, 1

a, 0

$$x(t) = \left(-\frac{t}{a} + 1\right)\left[u(t) - u(t-a)\right]$$

$$x(t) = -\frac{t}{a}\left[u(t) - u(t-a)\right] + u(t) - u(t-a)$$

$$= \frac{-1}{a}\left[\frac{1 + e^{-as}(1+as)}{s^2}\right] + \frac{1}{s} - \frac{e^{-as}}{s}$$

ROC: $\sigma > 0$